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TITLE OF THESIS

FARM ENTERPRISE SELECTION IN A
RISKY ENVIRONMENT

DEGREE FOR WHICH THESIS WAS PRESENTED MASTER OF SCIENCE

YEAR THIS DEGREE GRANTED FALL, 1982

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THE UNIVERSITY OF ALBERTA

FARM ENTERPRISE SELECTION IN A RISKY ENVIRONMENT

by



DALE ALEXANDER KALIEL

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE
OF MASTER OF SCIENCE

IN

AGRICULTURAL ECONOMICS

DEPARTMENT OF RURAL ECONOMY

EDMONTON, ALBERTA
FALL 1982

THE UNIVERSITY OF ALBERTA
FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled

FARM ENTERPRISE SELECTION IN A RISKY ENVIRONMENT

submitted by DALE ALEXANDER KALIEL in partial fulfilment of the requirements for the degree of MASTER OF SCIENCE IN AGRICULTURAL ECONOMICS

DEDICATED TO MY FAMILY

ABSTRACT

A general concern expressed regarding contemporary agriculture at the farm firm level is that variability and instability in returns is becoming relatively more prevalent. As a result, more thorough analyses of farm operating plans are necessary to assist in the achievement of farm goals and to accommodate the farm manager's risk attitudes. This thesis addresses the farm operating plan analysis problem by presenting a decision making tool adapted to assist in the selection of enterprises in an uncertain environment.

The thrust of the thesis is twofold: firstly, to illustrate the theoretical basis behind enterprise selection under uncertainty and relate these principles to a practical budgeting tool applicable at the farm level; secondly, to develop a matrix of correlation coefficients for an area of South Central Alberta in support of (and necessary for) the budgeting package. These efforts are intended as a pilot study, laying the groundwork for more extensive future research and development.

The "risk budgeting" decision tool is designed to employ a farm manager's subjective estimates of prices and yields for each enterprise. Given estimates of variable costs and the intended activity levels of the enterprises comprising a proposed operating plan, the program produces estimates of the expected gross margin and the variability (standard deviation) of the gross margin for that plan. Through the incorporation of correlation coefficients in the planning tool, the variability inherent in combinations of enterprises can be

evaluated, presenting an opportunity to assess risk reduction and diversification attempts. Upon obtaining a set of alternative enterprise combinations, the farm manager then selects the "best" operating plan by evaluating the risk-expected income trade-offs in accordance with his risk attitudes.

The major research effort of the study deals with the development of a matrix of correlation coefficients (of gross margins) among crop enterprises. Thirteen cropping alternatives are employed over a study area divided into seven sub-regions (municipalities). The crop coefficients in each of the sub-regional matrixes are subjected to a significance testing process which organizes them into homogeneous coefficient groupings. The resulting set of matrices is then in a suitable format for use in the risk budgeting package.

Significant opportunities for further research and development are identified. The applicability of the risk budgeting program could be enhanced through the expansion of the matrix to include a variety of other enterprises and a more extensive coverage of the province.

ACKNOWLEDGEMENTS

The process of completing a Master of Science program has taken a period of time over which I have come into contact, both socially and professionally, with a number of new faces. This process and these people have certainly served to enrich my knowledge and my life. Through this brief passage I wish to acknowledge a number of individuals and groups who have contributed to the furthering of my education, assisted in my research, and generally made this period of my career and my life one which I can reflect on as a time of happiness and fulfillment.

Firstly, I would like to acknowledge the assistance and cooperation of the Regional Business and Economic Services Branch (Alberta Agriculture), in particular Mr. Sandy Lauder from the Edmonton Office, and Mr. Lorne Owen and Mr. George Monner from the Airdrie Regional Office. Through their joint Alberta Agriculture-University of Alberta project, I was provided with an opportunity to combine the fulfillment of my thesis requirements with a topic I have grown quite close to over time.

Secondly, I would like to thank Mr. Rod Rains and Mr. Sid Selirio of the Alberta Hail and Crop Insurance Corporation who cordially and readily provided the data I required for my analyses.

My association with the Department of Rural Economy has been very fruitful and satisfying from the academic level, the technical research level and the personal interaction level. Of particular note are Mrs. Wendy Williamson, Ms. Evelyn Shapka, Mr. Jim Copeland and Mr. Clare

Shier who provided timely and competent technical assistance. In addition, my interaction with the academic staff and graduate students has served to be very rewarding.

Next, I would like to acknowledge the contributions of my thesis committee whose task it was to examine and assist in the fruits of my labor. The guidance, enthusiasm, and perseverance of my thesis supervisor, Dr. Len Bauer, has been greatly appreciated over the course of my program. The insights he has offered into the world of farm management will hold me in good stead for the rest of my career. As well, many thanks to Dr. Bill Phillips and Dr. Wayne Anderson who have provided sound and enlightening criticisms and suggestions.

Finally, I would like to express my gratitude to my friends and especially my family. They have been a source of encouragement to me, and at times have "rode herd" to stem my procrastination. Their patience and understanding has provided me with a feeling that cannot be fully expressed by words; words cannot describe the feeling of warmth for them in my heart.

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I. INTRODUCTION

During recent years the nature of farming has shifted toward a more commercial environment. This environment is characterized by technological change and increasing variability in returns. In response to this scenario, farm managers are, in general, becoming aware of the need to employ the principles of economics and financial management to ensure the viability of their operations.

In the parlance of economic theory, farm firms can be considered to be most closely associated with purely competitive, atomistic attributes. Characteristic of these firms is that they are price takers and quantity adjusters. Quantity adjustment at the farm firm level implies planning the most efficient use of resources in response to perceived market conditions. The farm manager assesses alternative resource allocation possibilities (operating plans) and selects the enterprise combination which most effectively moves him toward his goal.

However, difficulties arise in the assessment process. Since a period of time elapses between the actual commitment to (and execution of) an operating plan and the time the plan reaches fruition, many events can occur which alter the expected outcome. Uncertainty is introduced into the assessment of operating plans. It creates a problem in the decision of which enterprise combination is most suitable for the farm manager in achieving his goal.

This thesis focuses on a means by which farm managers can approach the problem of enterprise selection in an uncertain environment. To

introduce the topic, within this chapter the initial discussion addresses the farm management-decision making process interface followed by the problem statement. Then the objectives and scope of the study are detailed, and finally the outline and approach of the ensuing chapters is provided.

A. FARM MANAGEMENT AND THE DECISION MAKING PROCESS

The increasing need to employ business skills in farming provides an incentive for more exhaustive analyses of the type and specific nature of decisions to be made, and an active consideration of the decision making process. The term "farm management" encompasses these analyses and considerations. Farm management can be generally defined as:

"The art and science of making decisions about the use of available resources and acting on them, in an uncertain world, so that short term and long term goals of the farmer and his family are as fully satisfied as possible."¹

Doll, Rhodes and West (17, pg. 29) define farm management in the following manner:

"Management is the art of recognizing a problem, determining what to do about it, and doing it. The need for management arises because of uncertainty, that is, lack of knowledge about what the future will bring. Any given situation is called uncertain if the manager is not sure of the outcome at the time he must make the decision....

...the management of the farm business, perhaps because the owner is usually also its manager, is inextricably bound to the values and goals of the manager. Values are the manager's concept of "what ought to be". Goals are the ends to which he strives. Values determine the types of decisions made by managers."

These statements define the general means and motivation behind farm management, the pervasive question of "why?"

The definitions of farm management provided do not as yet yield much insight into the mechanics of management. They allude to the making of decisions regarding the resources and conditions prevailing upon the farm manager, and the goals and values he embraces. Without the effective use of the decision making process, the farm manager would be at a loss to attain his goals. The decision making process, or the functions of management, can be described as five sequential, yet not mutually exclusive steps. Doll, Rhodes and West (17, pg. 30) have stated the five functions of management as follows:

- "1. Gathering facts in order to learn new methods and identifying problems;
2. Analyzing facts to formulate alternative production plans;
3. Deciding on one of the production plans;
4. Putting the plan into action; and
5. Accepting the responsibility for the consequences."

Regardless of the degree to which the farm manager consciously employs the decision making process, the fact that the process is in use indicates recognition of, and adaptation to changing conditions. Changing conditions refer to technological change, agricultural policy change, variability in resource and product markets, and so on. In farm planning terms, these conditions range from evaluation of a single farm enterprise to a total farm plan, to the inclusion of off-farm concerns.

Upon stating the "how's" and "why's" involved in farm management, all that is necessary is an indication of direction, that is, to what

the decision making process is applied (to attain the ascribed goals). The "what's" are defined as "types of management decisions." The types of management decisions, as put forward by numerous authors, generally conform to the following list delineated by Herbst (21, pg. 1):

- "1. What to produce;
2. How much to produce;
3. The kinds and amounts of resources to use;
4. When and where to sell and buy, and
5. How to finance the operation."

These categories serve only to emphasize particular areas of economic rationale. They are important in terms of identifying and dealing with specific problem areas which may be only part of a more complex analysis.

Finally, note from the list of types of management decisions that management includes more than economic considerations. It deals, as well, with physical, sociological and psychological concerns. These concerns pertain to the characteristics and constraints of the farm manager's individual operation, his goal set, and how he responds to these restrictions, according to his knowledge and understanding of them.

B. THE PROBLEM

Both of the definitions given for farm management refer to making decisions regarding the allocation of scarce resources in an attempt to attain goals in a setting of uncertainty. Uncertainty is loosely defined as "[when] the manager is not sure of the outcome at the time he must make the decision" (17, pg. 29). The farm planning implications

of introducing uncertainty into the classification of types of management decisions reflect an increase in the complexity of decision making. Expected farm incomes, projected in farm plans formulated on the basis of certainty in resource and product components, are now subject to some degree of uncertainty. Variability arises from fluctuations in natural (physical) and/or market factors.

This variability, in conjunction with the general commercialization of farming, precipitates a situation of instability. Farm businesses have become increasingly sensitive to variation in returns. Farm managers must carefully assess their operating plans to ensure not only a movement toward goal fulfillment, but more basically, that the financial viability of the farm is not compromised. Analysis of a proposed operating plan now stresses an assessment of the likelihood that a level of returns, required to cover costs, will be achieved.

The problem of assessing variability in farm income (specifically arising from grain crop enterprise combinations) is addressed in this study through a planning tool compatible with the farm manager's information and needs. The analysis of variability of farm income, or the evaluation of riskiness involved in the total farm operation, is handled by farm managers in a subjective manner. Farm budgets are usually established on the basis of average (expected) yields, prices, and variable costs anticipated for the upcoming accounting period(s). Variability estimates can be derived from the same source as the average figures are obtained. By soliciting ranges over which the prices and yields are likely to vary from the farm manager, the initial requirements for computing a farm operating budget including an

estimation of riskiness are satisfied. The planning tool derives the expected returns and variability associated with each enterprise and, through a set of interaction components, summarizes the risks and returns inherent in a proposed farm operating plan.

The interaction components, mentioned above, are an integral part of the budgeting procedure. They represent the degree and direction to which enterprises are interrelated, that is, how income generated by one enterprise varies with respect to another. By including these interaction components in the budgeting procedure, it becomes possible to select enterprise groupings which combine to reduce risk, rather than on the basis of their individual lower-risk characteristics.

A secondary problem arises in addressing the initial problem of providing a planning tool to aid farm managers in dealing with risk. As noted previously, farm managers can indirectly arrive at subjective estimates of variability of returns to individual enterprises. However, it is highly unlikely that they would be able to derive the interaction components for all of the available enterprise alternatives in the same manner. The problem of deriving these interaction components in order to confront the initial, general problem is addressed specifically in this thesis.

C. OBJECTIVES AND SCOPE OF THE STUDY

The objectives of this thesis are two-fold. Firstly, expand upon a budgeting procedure to evaluate the expected returns and variability inherent in individual operating plans. Secondly, develop a series of

interaction components (correlation coefficients) of gross margins among an array of crop enterprise alternatives. Fulfillment of these objectives should provide a practical means for farm enterprise selection in a risky environment.

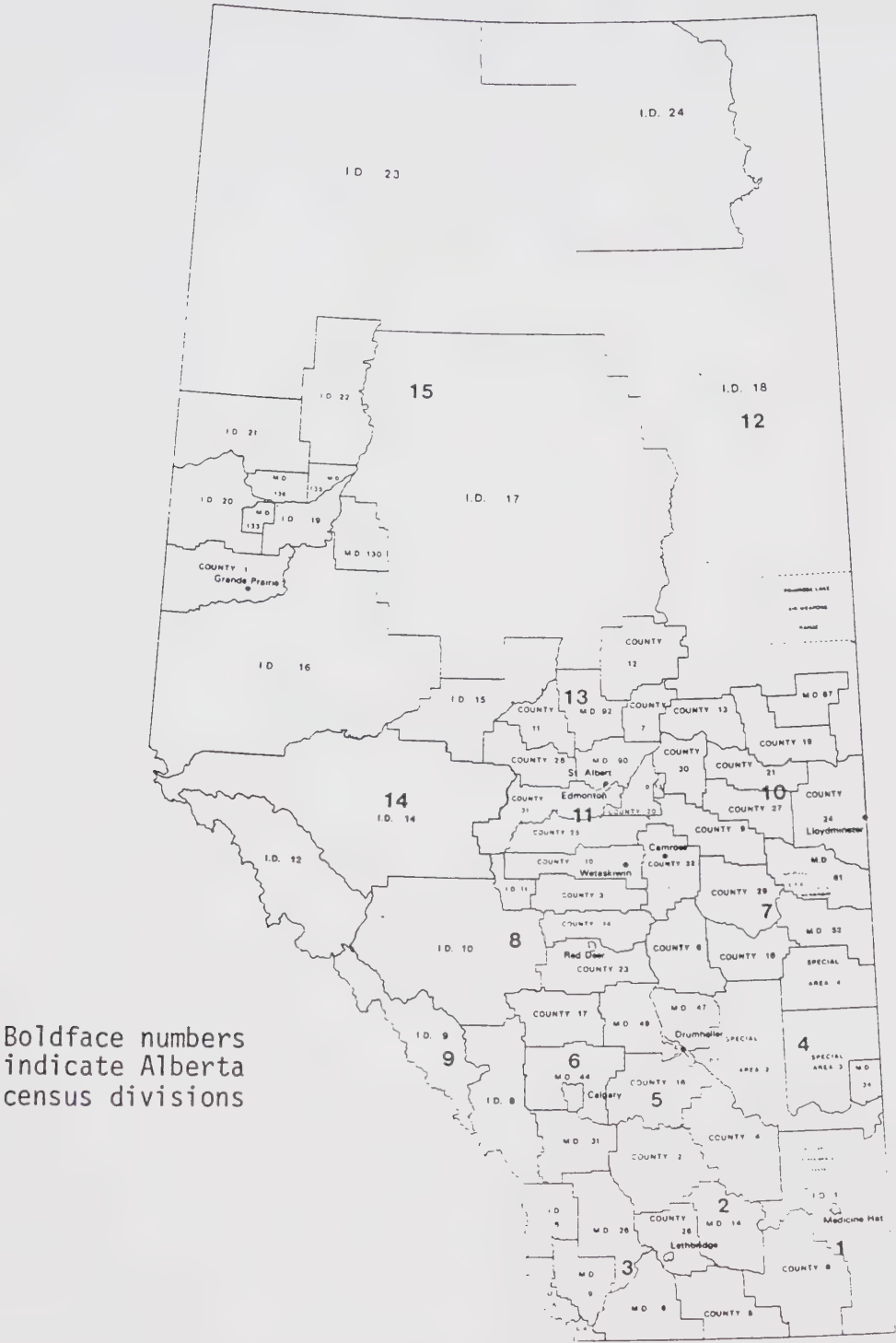
The research thrust of the thesis is to develop a matrix of correlation coefficients among major cropping enterprises for an area of South-Central Alberta (Alberta Agriculture Extension Region II). The study area, shown in Figure 1.1, extends from the mountains on the west to the Saskatchewan border on the east, encompassing a wide variety of agroclimatic conditions. This research is conducted as a pilot study, documenting a procedure to obtain coefficients of a significant nature.²

The "risk budgeting" procedure is discussed in terms of the theoretical considerations and its application. The discussion expands upon the application of the pertinent economic theory to decision making in farm management. As well, an exposition of the statistical relationships underlying the budgeting procedure is included. The intention of these explanations is to indicate the role the risk budgeting procedure can perform as a farm management tool, and to provide an understanding of the statistical concepts involved.

D. STUDY OUTLINE

The study proceeds in Chapter II with a discussion of the theoretical considerations. Chapter III gives an outline of the methodology and data collected. In Chapter IV, the analyses and results are

Figure 1.1
The Study Area



Source: Alberta Transportation, Mapping Division, "Alberta Resource Maps", November 1978.

presented. Finally, Chapter V contains a summary and discussion of the findings of the study, suggestions for expansion of the correlation matrix (to include other enterprises), and a review of uses for the risk budgeting procedure.

ENDNOTES

1. Paraphrased from Vincent, W.H., Economics and Management in Agriculture, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1962.
2. Significance, in this sense, implies common crop groupings and interregional similarities across the study area.

II. THEORETICAL CONSIDERATIONS

The enterprise selection decision has its basis in economic theory. For the farm manager, it addresses the problems of what to produce and how much to produce. These problems can be analyzed and evaluated through the partial budgeting procedure. The format of this chapter is firstly, to outline a group of feasibility conditions which constitute the background information necessary for budgeting. Then, the economic rationale (from the theory of production economics) pertaining to the budgeting procedure is presented. The discussion on the theory of production is separated into a description of decision making under certainty, and then moves on to detail additional concerns when uncertainty is introduced into the process. The discussion of decision making under certainty is composed of the derivation of the economic decision rules (i.e., the marginal conditions), and their application to the partial budget. The discussion of the uncertainty case expands upon (1) the effects of variability upon the certainty case; (2) risk attitudes and utility; (3) decision making under uncertainty; and (4) the framework of the risk budgeting procedure. This chapter should provide an insight into enterprise selection under risk and how the risk budgeting procedure can aid in the decision making process.

A. FEASIBILITY CONDITIONS

The definitions of farm management provided in Chapter I refer to decision making under risk¹ and its role in goal fulfillment. The

decision making procedure is further detailed as a stepwise, continuous process. This process may be viewed as a means by which the feasibility constraints of the farming operation are implicitly recognized. The feasibility constraints are categorized as follows: (1) technical feasibility; (2) economic feasibility; and (3) human feasibility. The first two constraints impose restrictions through physical and financial factors and encompass the sources of variability in returns. The third constraint, human feasibility, imposes restrictions on enterprise combinations through risk attitudes and personal preference.

Technical feasibility refers to the technical input-output relationships, that is, the combination of factors employed in production and their interrelationships. The factor-factor, factor-product, and product-product interactions reflect biological, climatic, and physical (technological) considerations.

Economic feasibility refers to the general profitability in terms of the price and cost relationships associated with each enterprise and activity level. These constraints are directly linked with the technical relationships in that they provide a means to evaluate the relative profitability of enterprises and operating plans.

The human feasibility constraints express the farm manager's personal preferences and risk attitudes in accordance with his goals. Firstly, although an enterprise may be both technically and economically feasible for an individual operation, the farm manager's personal preference may dictate that the enterprise is not considered as an alternative. Secondly, the farm manager's risk attitudes, i.e., his willingness to assume risk, may act to restrict enterprises selected

for the operating plan. In other words, the farm manager may consider the risk he perceives (arising from the technical and economic relationships) substantial enough to jeopardize the financial viability of the farm and/or the goals that have been set.

The combination of the three feasibility constraints define the framework within which the farm manager conducts his decision making. The biological, climatic, and physical relationships, and the factor and product market considerations are evaluated with respect to the expected returns and inherent variability. A broad spectrum of methods are available to farm managers to deal with risk, including discounting future returns, maintaining a contingency reserve, production insurance, maintaining flexibility in enterprise alternatives and physical facilities and so on. The planning tool presented in this thesis focuses on one of these methods, a process to assess the effects and benefits of diversification of enterprises within proposed operating plans. This tool is designed to aid farm managers in quantifying the risk and returns associated with proposed operating plans (re: technical and economic conditions) such that they are evaluated regarding the manager's human feasibility constraints.

B. THE CERTAINTY CASE

The traditional case of production under certainty, as outlined in the theory of production in a purely competitive market, applies directly to the technical and economic feasibility constraints. The short run enterprise selection decision, couched in this theory, is

bound by a number of assumptions. These assumptions provide the framework in which the production decision can be illustrated without complicating factors (other things remaining equal). The major assumptions connected with the short run enterprise selection decision are (1) perfect knowledge of factor and product prices exist; (2) perfect knowledge of the factor-product technical relationships exists; (3) participants are price takers and quantity adjusters; and (4) participants pursue the goal of profit maximization. Under these assumptions, the farm manager selects enterprises which, according to their technical and economic feasibility characteristics, will maximize profit for his farm operation. In other words, the farm manager allocates his scarce resources, according to a set of decision rules, in order to attain his goal.

These "decision rules" can be derived from a production function, a mathematical expression of the economic and technical relationships (12, pg. 124-131; 6, pg. 10-13),

$$F(y_1, \dots, y_n; x_1, \dots, x_m) = 0 \quad (2.1)$$

where F is the production function, stated implicitly as a function of n products and m inputs. The first step in obtaining the decision rules is to establish the mathematical expression of the profit function and then maximize (Max.) the function subject to (s.t.) its constraints. Therefore,

$$\text{Max. } Y = \sum_{i=1}^n p_i y_i - \sum_{j=1}^m r_j x_j$$

$$\text{s.t. } F(y_1, \dots, y_n; x_1, \dots, x_m) = 0 \quad (2.2)$$

$$y_i \geq 0 \quad i = 1, \dots, n$$

$$x_j > 0 \quad j = 1, \dots, m$$

where

Y = the profit function,

y_i = the level of the i th output

x_j = the level of the j th input

p_i = price per unit of the i th output

r_j = cost per unit of the j th input, and

F = the production function, stated in implicit form.

To maximize the profit function, it is necessary to form the Lagrange function,

$$R(y, x, \lambda) = \sum_{i=1}^n p_i y_i - \sum_{j=1}^m r_j x_j + \lambda [F(y_i; x_j)] \quad (2.3)$$

where λ is the Lagrange multiplier. To solve the Lagrange function, the partial derivatives with respect to y, x , and λ are taken, set equal to zero, and solved simultaneously. The resulting partial derivatives are

$$\begin{aligned} \partial R / \partial y_i &= p_i + \lambda \partial F / \partial y_i = 0, \quad i = 1, \dots, n \\ \partial R / \partial x_j &= -r_j + \lambda \partial F / \partial x_j = 0, \quad j = 1, \dots, m \\ \partial R / \partial \lambda &= F(y_i; x_j) = 0 \end{aligned} \quad (2.4)$$

The solution of this system of simultaneous equations leads to the three basic decision rules.² All three of these conditions must be met for profit to be maximized. Each decision rule is stated in its

mathematical form and then interpreted. Firstly,

$$p_i \, dy_i / d_{xj} \geq r_j \quad (2.5)$$

or, the Value of the Marginal Product (VMP) of any factor must equal its Marginal Factor Cost (MFC). Alternatively, as long as the added return to a factor exceeds its added cost, additional units of the factor should be employed until, in the limit, VMP must equal MFC.

Secondly,

$$-dx_j/dx_s \geq r_s/r_j \quad (2.6)$$

or the Marginal Rate of Substitution (MRS) of one factor for another should equal the ratio of their respective costs.³ Alternatively, as long as the decreased costs from the factor being replaced exceed the increased cost of the factor being added, then the substitution should continue until, in the limit, the MRS equals its price ratio. Thirdly,

$$-dy_i/dy_k \leq p_k/p_i \quad (2.7)$$

or, at the point of profit maximization, the Marginal Rate of Transformation (MRT) between any two products must equal their ratio of respective prices. In other words, as long as the added returns from increased production of one product exceed the decreased returns from the decline in production of another, then the exchange of one product for another should continue until, in the limit, the MRT equals its price ratio. Equations 2.5, 2.6, and 2.7 will be referred to as Decision Rule #1 (DR 1), Decision Rule #2 (DR 2), and Decision Rule #3 (DR 3) respectively in the subsequent discussions.

A commonly used tool in farm planning is the partial budget. A typical partial budget format is displayed in Figure 2.1. The relationship between the decision rules and the partial budget can be

Figure 2.1
The Partial Budget

| | |
|---|--|
| <p>1.) <u>Added Costs:</u> \$ _____</p> <p>_____</p> <p>_____</p> <p>_____</p> <p>_____</p> <p style="text-align: right;">Total _____</p> | <p>4.) <u>Added Receipts:</u> \$ _____</p> <p>_____</p> <p>_____</p> <p>_____</p> <p>_____</p> <p style="text-align: right;">Total _____</p> |
| <p>2.) <u>Reduced Receipts:</u></p> <p>_____</p> <p>_____</p> <p>_____</p> <p>_____</p> <p style="text-align: right;">Total _____</p> | <p>5.) <u>Reduced Costs:</u></p> <p>_____</p> <p>_____</p> <p>_____</p> <p>_____</p> <p style="text-align: right;">Total _____</p> |
| <p>3.) <u>Total Reduced Income:</u></p> <p>(Add Total 1 to Total 2) A _____</p> | <p>6.) <u>Total Increased Income:</u></p> <p>(Add Total 4 to Total 5) B _____</p> |
| <p>NET CHANGE IN INCOME: \$ </p> <p style="margin-left: 150px;">(B-A)</p> | |

Source: Couture, M.J., Farm Business Management, 2nd ed., MacDonald Campus of McGill University, 1978.

illustrated by minor manipulations of the mathematical forms of the decision rules. The mathematical statements for this discussion are in the form of discrete interval estimates (denoted by " Δ "). DR 1 is stated as

$$p_i \Delta y_i \geq r_j \Delta x_j,$$

or, added returns \geq added costs. This relationship appears in the partial budget as shown in Figure 2.2. DR 2 is placed in the partial budget format in Figure 2.3. Equation 2.6 is manipulated to take the format

$$-r_j \Delta x_j \geq r_k \Delta x_k,$$

or, reduced costs \geq added costs. Therefore, as long as the reduced costs are greater than the added costs, other things remaining constant, the proposed change should be carried out. The remaining decision rule, DR 3, can be placed in the partial budget when put in the following form:

$$-p_i \Delta y_i \leq p_k \Delta y_k,$$

or, reduced revenue \leq added revenue, as shown in Figure 2.4. It implies that as long as the added revenue from a proposed change is greater than the reduced revenue, then the change should be undertaken.

From this discussion, the connection between the partial budget and economic theory, as presented by Hicks (22, pg. 78-82; 18, pg. 9), becomes evident. The partial budgeting format presented has been shown to apply to the case where all of the assumptions for production under certainty hold. The perspective of the budgeting format is now extended to accommodate the uncertainty case. When the frame of

Figure 2.2
Partial Budget and DR 1

| | |
|---------------------------------|------------------------------------|
| Added Costs $r_j \Delta x_j$ | Added Receipts $p_i \Delta y_i$ |
| Reduced Receipts | Reduced Costs |

Figure 2.3
Partial Budget and DR 2

| | |
|---------------------------------|-----------------------------------|
| Added Costs $r_k \Delta x_k$ | Added Receipts |
| Reduced Receipts | Reduced Costs $r_j \Delta x_j$ |

reference changes to represent an "actual" decision making situation, adjustments in the manner in which the theory is presented must be made for incorporation into the partial budget. These adjustments are necessary due to intangible items, or items which are difficult to quantify.

Figure 2.4
Partial Budget and DR 3

| | |
|--------------------------------------|------------------------------------|
| Added Costs | Added Receipts $p_k \Delta y_k$ |
| Reduced Receipts $p_i \Delta y_i$ | Reduced Costs |

The item perhaps most difficult to quantify is the result of relaxing the perfect knowledge assumptions. With the certainty case, decision making dealt only with the technical and economic feasibility constraints. When the perfect knowledge assumption is relaxed, causing a move to the realm of uncertainty, the third feasibility constraint, human feasibility, becomes a much more active and important variable in the decision making process.

C. THE UNCERTAINTY CASE

The assumptions used in describing the certainty case indicate that factor and product prices, and the technical input-output relationships are known entities. When the perspective is changed to the uncertainty case, these elements take on stochastic characteristics, i.e., each component is characterized by a probability distribution. This change adds another dimension to the farm manager's goal achievement problem (previously to maximize profits). The returns received from employing a bundle of inputs (i.e. an operating plan) can vary in accordance with the probability distributions of the component technical and economic relationships. Associated with each operating plan, there is a probability of incurring a loss. The reaction of the decision maker would then be to attempt to select enterprises which he feels yield a reasonable certainty of maintaining the viability of the farm unit. This selection process involves the incorporation into the decision making procedure of the farm manager's subjective notions of the probability distributions confronting him.

RISK ATTITUDES AND UTILITY

The decision on the bundle of enterprises selected for the operating plan hinges on the farm manager's perception of the cumulative variability inherent in the plan. In conjunction with these perceptions, the alternatives are scrutinized subject to his attitudes toward risk. The argument has its basis in utility theory which will be used to illustrate the mechanics of decision making under risk.

Von Neumann and Morgenstern (28) present a concept of cardinal utility which deals with a "preference ranking of risky alternatives" (6, pg. 24). Chernoff and Moses (10, pg. 82) discuss a number of assumptions necessary in the development of a utility function for income. The properties of this utility function specify that its first derivative is always positive and its second derivative may be negative, zero or positive, depending upon the risk attitudes of the decision maker.⁴ The categories of risk attitudes corresponding to the signs of the second derivatives are risk averseness, risk neutrality and risk preference, respectively.

In accordance with the properties of the utility function, its derivatives have implications with respect to stochastic dominance. First order stochastic dominance arising from the positive first derivative (the monotonic property of the utility function), implies more expected income is preferred to less in each risk category (4, pg. 282). Second order stochastic dominance pertains only to the situation of risk averseness up to the point of neutrality, and involves the trade-off between variance and expected income. This trade-off is evaluated in terms of the probability of attaining a given (range of) expected income (4, pg. 284). At a later point in this chapter, the role of stochastic dominance in the decision making process is illustrated.

A process of Taylor series expansion allows the previously described utility functions to be expressed in expected value-variance (E-V) space as expected utility functions (6, pg. 25; 4, pg. 97). Figures 2.5, 2.6 and 2.7 display the indifference, or iso-utility

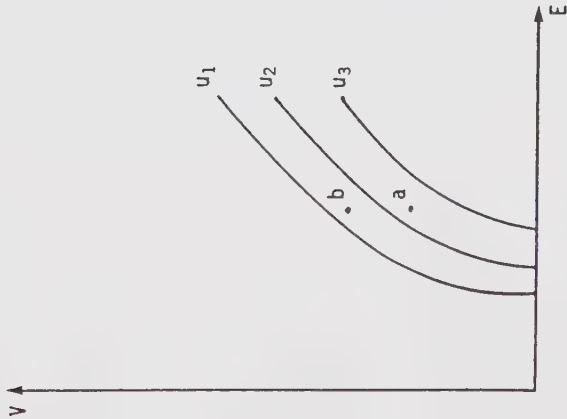


Figure 2.5: Indifference curves for an individual who is a risk averter.

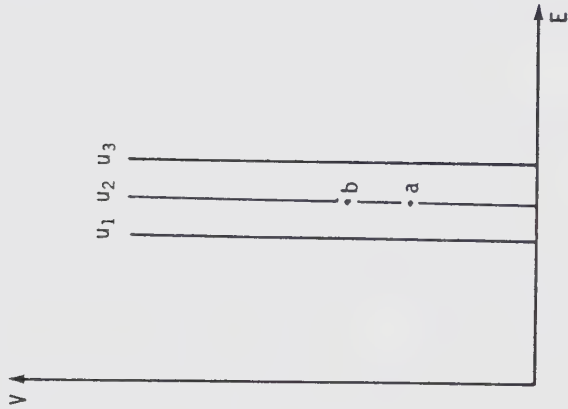


Figure 2.6: Indifference curves for an individual who is risk neutral.

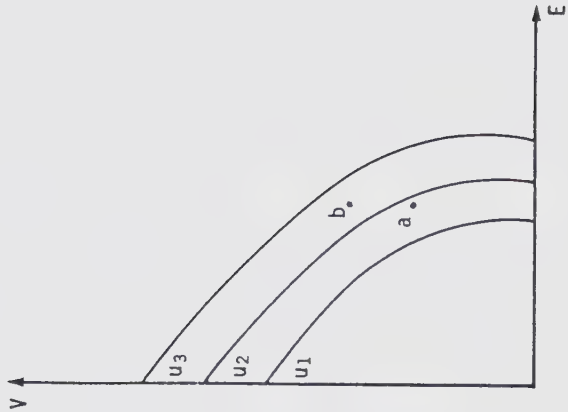


Figure 2.7: Indifference curves for an individual who prefers risk.

curves in E-V space corresponding to the three risk categories. The shape of the indifference curves dependent upon the individuals' marginal utility for income (decreasing, constant, or increasing). The characteristics of the indifference curves are as follows (6, pg. 28-30):

"The family of indifference curves has the following characteristics:

1. For any two alternatives, each with the same variance, the one with the higher expected income will yield the greater expected utility.
2. For any two alternatives, a and b, each having the same expected income:
 - (a) where the marginal utility of income is increasing the alternative with the greater variance will yield the greatest expected utility as shown in Figure [2.7]
 - (b) where the marginal utility of income is decreasing the alternative with the lower variance will yield the higher expected utility as shown in Figure [2.5]
 - (c) where the marginal utility of income is constant both alternatives will have the same expected utility as shown in Figure [2.6]."

These three types of indifference curves encompass the factors involved in individuals' attitudes toward risk and are key elements in the human feasibility constraints. Although there is a definite occurrence of risk preferring individuals (15, pg. 31), in general, people have a tendency toward risk averseness (4, pg. 89). In addition, the nature of enterprise selection differs for the risk preferring individuals. Therefore, from this point forward, the major emphasis is placed on the discussion of the case of risk averseness (up to the point of neutrality).

The aim of this section is to provide an insight into the means by which the human feasibility constraints are incorporated into the decision making procedure. This insight into the theoretical framework of utility sets the groundwork for the discussion on the evaluation of risky alternatives (enterprises selection) which deals with the technical and economic feasibility constraints.

DECISION MAKING UNDER UNCERTAINTY

The discussion on utility and risk attitudes serves to highlight the expected income and variance components as decision criteria. In this section the discussion initially delineates the aggregated mathematical components of the technical and economic constraints with respect to the expected income-variance criteria. Once these are depicted graphically within E-V space, the enterprise selection solution is illustrated. Finally, there is an exposition of the effects of varying one of the major components within the aggregated relationship, the correlation coefficient.

The relationships identifying alternative operating plans in E-V space (6, pg. 127) have their basis in portfolio analysis theory. The expected income for a combination of enterprises, $E(Y)$, is represented by

$$E(Y) = \sum_{i=1}^n \mu_i y_i \quad (2.8)$$

where μ_i = the expected income per unit of the i th activity, and
 y_i = the activity level of the i th enterprise.

The expression for the variance of income from a combination of enterprises, $V(Y)$, is:

$$V(Y) = \sum_{i=1}^n \sigma_i^2 y_i^2 + 2 \sum_{i=1}^n \sum_{j<i}^n r_{ij} \sigma_i \sigma_j y_i y_j \quad (2.9)$$

where σ_i^2 = the variance of income per unit of the i th activity, and r_{ij} = the correlation coefficient between the incomes from the i th and j th activities.

Therefore, each combination of enterprises (at specified activity levels) can be expressed as an income-variance pair. There are a multitude of possible enterprise combinations and activity levels, from which the farm manager must select the "best" one. Figure 2.8 illustrates a few such hypothetical operating plans in E-V space.

The first phase in eliminating farm plans as poor alternatives is through the concept of stochastic dominance (6, pg, 41). As noted previously, for the decision maker to maximize utility, when faced with a number of alternatives of the same variance, he will select the alternative with the highest expected income (or, similarly, when faced with a number of alternatives with the same expected income, he will select the option with the lowest variance). Stochastic dominance, in relation to the operating plans presented in Figure 2.8, implies that any enterprises occupying the north-west quadrant of another are dominated, i.e., less efficient. This situation is illustrated in Figure 2.9. Point a dominates point b; c dominates both b and d; and point e dominates d. Points a, c and e are not dominated by any of the operating plans displayed. From this figure, the set of efficient points (operating plans) is observed to occupy positions to the extreme right.

Figure 2.8
Operating Plan Income-Variance Pairs

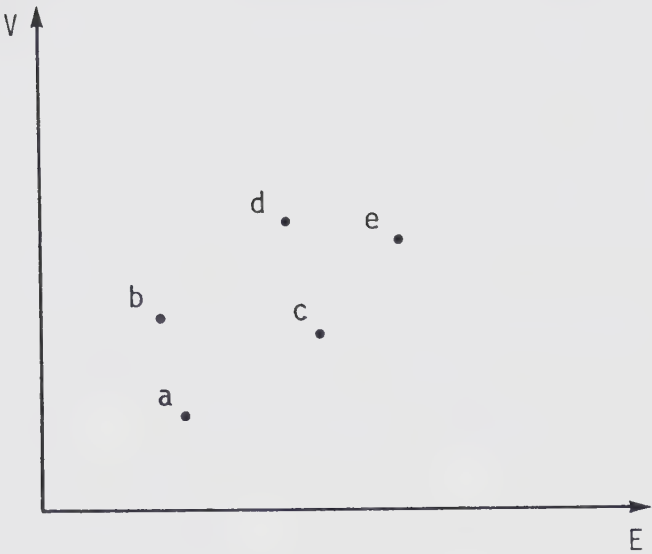
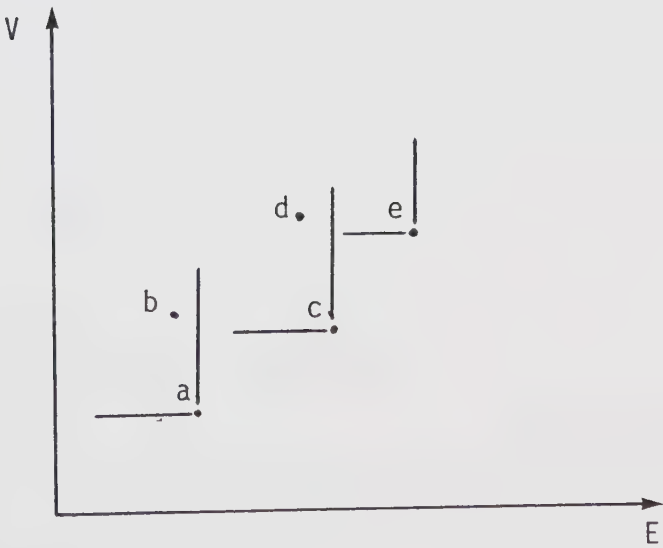


Figure 2.9
An Illustration of Stochastic Dominance



If quadratic programming procedures were used, a continuous function of risk efficient points could be determined (6; 4). Figure 2.10 displays the locus of risk efficient points, termed an "efficiency frontier". In order to conceptually arrive at the operating plan which the decision maker will select, it is necessary to re-introduce the set of indifference curves described in the previous section. Figure 2.11 displays the efficiency frontier and the family of indifference curves for the (assumed typical) risk averse decision maker. The operating plan corresponding to the resulting tangency, point m, will be selected by the farm manager. It embodies the combination of enterprises which conform to the farm manager's requirements for a risk-expected income trade-off, i.e., in accordance with his risk attitudes.

The risk budgeting technique used in this thesis is not a minimizing one such as would be derived through quadratic programming or through Minimization of Total Absolute Deviations (MOTAD) programming. Rather it calculates the expected income and variance of particular enterprise combinations. As such, the combinations evaluated will not be risk efficient in the more sophisticated risk programming sense, however, they will be combinations which the operator himself feels are operationally feasible.

IMPACT OF THE CORRELATION COEFFICIENT

An analysis of the combination of technical, economic, and human feasibility constraints comprising an individual farm unit leads to a decision as conceptualized in Figure 2.11. Given a set of human

Figure 2.10
The Efficiency Frontier

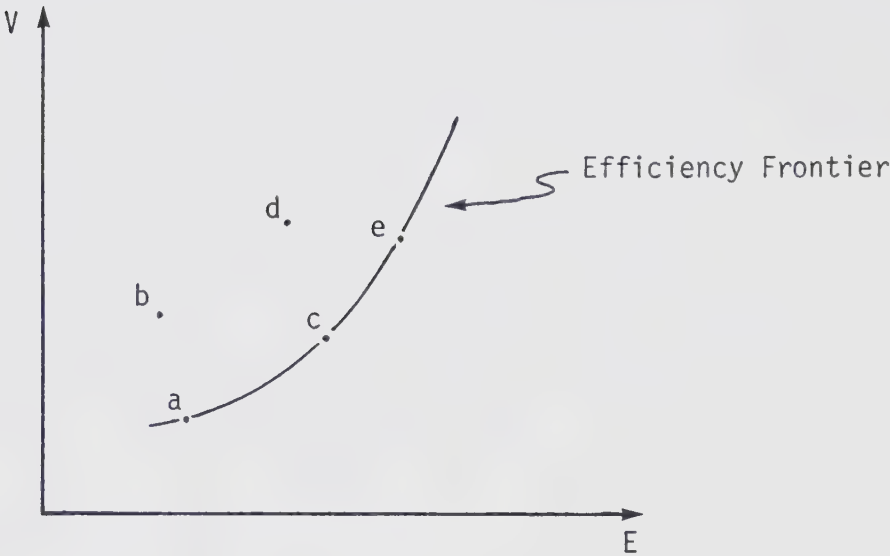
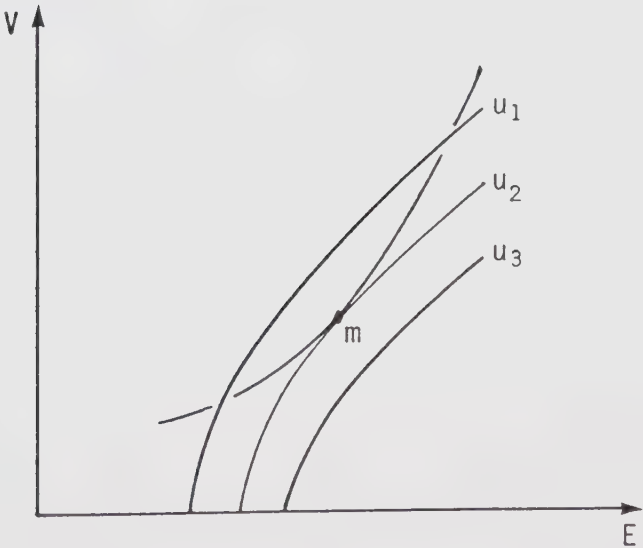


Figure 2.11
The Optimal Enterprise Combination Choice



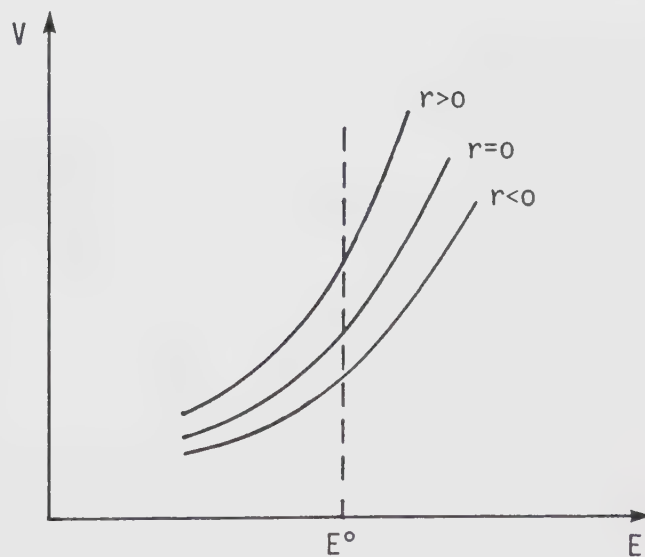
feasibility constraints, the farm manager will select a combination of enterprises from a group of risk efficient plans in order to maximize his utility for income. However, within the mathematical formulation of the efficiency frontier, the correlation coefficient component plays a vital role in the positioning of the frontier in E-V space and, consequently, the operating plans selected.

Equation 2.9 expressed the variance for an individual operating plan. The magnitude of this variance component can be altered by the sign and magnitude of the correlation among activities within the operating plan. Figure 2.12 illustrates the effect of the correlation coefficient on the efficiency frontier. It assumes that the bulk of the enterprises in the operating plans are positively correlated, uncorrelated, or negatively correlated. If the enterprises are more positively correlated, the efficiency frontier pivots in a counter-clockwise fashion so that for a particular expected income, a higher variance is encountered. The opposite effect occurs if the enterprises are more negatively correlated. When the family of indifference curves for any of the risk attitude categories are super-imposed upon Figure 2.12, the outcomes become apparent. This point is clarified, from an intuitive point of view, by referring back to Equation 2.9. If two activities are negatively correlated, then a combination of the two have a reduced total variance for a particular level of expected income, as compared to the case where they are uncorrelated. Conversely, if they are positively correlated, then their total variance is greater than the uncorrelated case.

The effect of varying degrees of correlation within alternative operating plans has implications with respect to diversification.⁵ Although the correlation relationship affects all three risk attitude categories, the more common, and perhaps most important responses arise from the case of risk averseness. As cited by Chong et al (11; pg. 1), Tobin (26) noted that diversification occurs as a response by the risk averse individuals to decrease the variability inherent in an operating plan. The response of the risk natural and risk taking individuals will tend toward specialization.

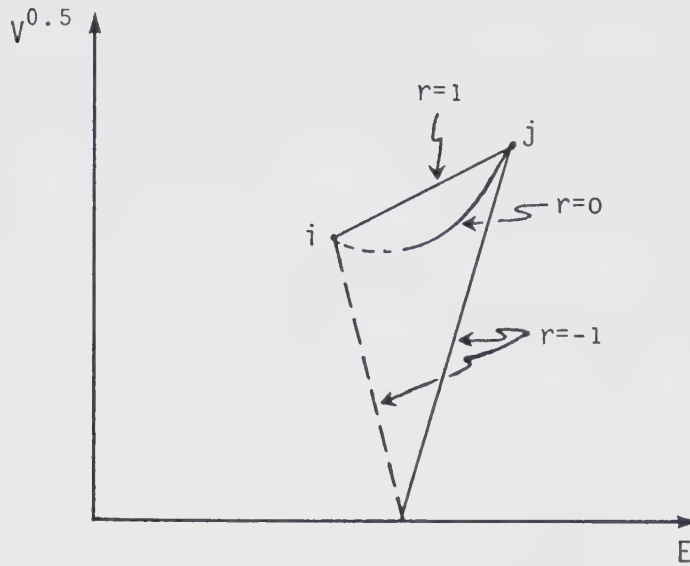
Figure 2.12

Effect of the Correlation Coefficient on the Efficiency Frontier



Anderson, Dillon, and Hardaker (4, pg. 193) discuss the relationship between correlation and diversification for a two alternative case. Their discussion outlines the relation presented in Figure 2.13. The mean, variance and correlation coefficient determine the shape and skewness of the resulting curves. The horizontal perspective is a

Figure 2.13
Relation between Correlation and Diversification

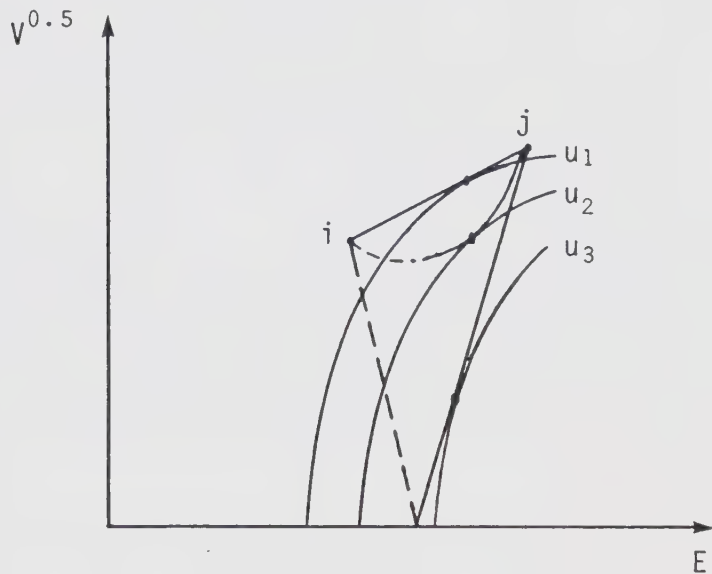


function of the relative difference between the expected returns from each enterprise (i.e., the greater the difference between the expected returns for alternatives i and j , the greater the horizontal distance separating them on the graph, holding the variances constant). The vertical perspective, or the skewness of the figure, reflects the relative difference of the variances of each of the risky prospects. The impact of the correlation coefficient is illustrated by the shapes of the three different curves in the figure below (a detailed example is provided in Appendix 3). These shapes represent three values, from a continuum of $+1$ to -1 , which the correlation coefficient can assume. With respect to the risk attitude categories (types of indifference curves), the risk taker and risk neutral individual would specialize in risky prospect j , regardless of the correlation. However, it is most likely that the risk averse individual would select a combination of

the risky alternatives, i and j as shown in Figure 2.14. In this case, the level of maximum utility would tend to increase as the correlation coefficient becomes less positive, and then more negative. Consequently, it is expected that a risk averse individual would attempt to diversify his enterprise combination, particularly including more negatively correlated enterprises.

Figure 2.14

The Effect of Correlation on Enterprise Combination



D. THE RISK BUDGETING PROCEDURE

The previous discussion outlines the factors involved in enterprise selection and the manner in which the selection decision is achieved. The risk budgeting procedure is presented as a planning tool through which the farm manager quantifies the expected income and variance for alternative operating plans. The risk-income pairs are then

evaluated according to the farm manager's human feasibility constraints to select a desirable operating plan.

The expected income and variance relationships expressed in Equations 2.8 and 2.9 respectively, are the "cornerstone" equations in the risk budgeting procedure. The procedure is tailored to the use of the farm manager's subjective estimates of prices and yields as input to these equations. This section deals with the mechanics of achieving the estimates of expected income and variance.

Gross margin is used to represent expected income in the risk budgeting procedure. It is defined as gross revenue less variable costs. Gross revenue is expressed as the product of physical yield and the market price. Variable costs are those production costs which can be employed at varying levels of application (in the relevant time frame). For the purposes of the risk budgeting procedure, variable costs are assumed non-stochastic⁶ and fixed costs are not considered at this level of the production decision. Gross margin can be stated mathematically as:

$$GM_i = y_i q_i p_i - y_i c_i = y_i (q_i p_i - c_i) \quad (2.10)$$

where

GM_i = the gross margin contributed by the i th enterprise,

y_i = the activity level of the i th enterprise,

q_i = yield per unit of the i th activity,

p_i = price per unit yield of the i th activity,

and

c_i = variable cost per unit of the i th activity.

This represents the contribution by the i th enterprise to the total gross margin of the business.

The yield and market price are assumed to be stochastically independent. This does not contradict the nature of the competitive market faced by farm managers in that they can be considered as price takers and quantity adjusters only. Proceeding from this assumption, the following relationships arise:

$$E(GM_i) = y_i [E(q_i p_i) - c_i] \quad (2.11)$$

$$= y_i [E(q_i p_i) - E(c_i)]$$

$$E(GM_i) = y_i [E(q_i)E(p_i) - E(c_i)] \quad (2.12)$$

where

$E(GM_i)$ = the expected gross margin of the i th activity,

$E(q_i)$ = the expected yield per unit of the i th activity,

$E(p_i)$ = the expected price per unit yield of the i th activity, and

$E(c_i)$ = the expected variable cost per unit of the i th activity.

Given that variable costs are assumed non-stochastic, Equation 2.12 reduces to

$$E(GM_i) = y_i [E(q_i)E(p_i) - c_i]$$

which can be stated as

$$E(GM_i) = y_i \mu_i \quad (2.13)$$

where μ_i is the expected gross margin per unit of the i th activity.

The variance relationship⁷, on the enterprise level is expressed as

$$V(GM_i) = V(q_i)V(p_i) + V(q_i) [E(p_i)]^2 + V(p_i) [E(q_i)]^2 \quad (2.14)$$

where

$V(GM_i) = \sigma_i^2$, the variance of the gross margin per unit of the i th activity,

$V(q_i)$ = variance of the yield per unit activity, and

$V(p_i)$ = variance of the price per unit yield of the i th activity.

These enterprise level relationships are extended to a total farm basis as follows:

$$E(GM_T) = \sum_{i=1}^n y_i \mu_i \quad (2.15)$$

$$V(GM_T) = \sum_{i=1}^n \sigma_i^2 y_i^2 + 2 \sum_{i=1}^n \sum_{j<i}^n r_{ij} y_i y_j \sigma_i \sigma_j \quad (2.16)$$

where

$E(GM_T)$ = the expected gross margin for the total farm operation,

$V(GM_T)$ = the variance of the gross margin for the total farm operation, and

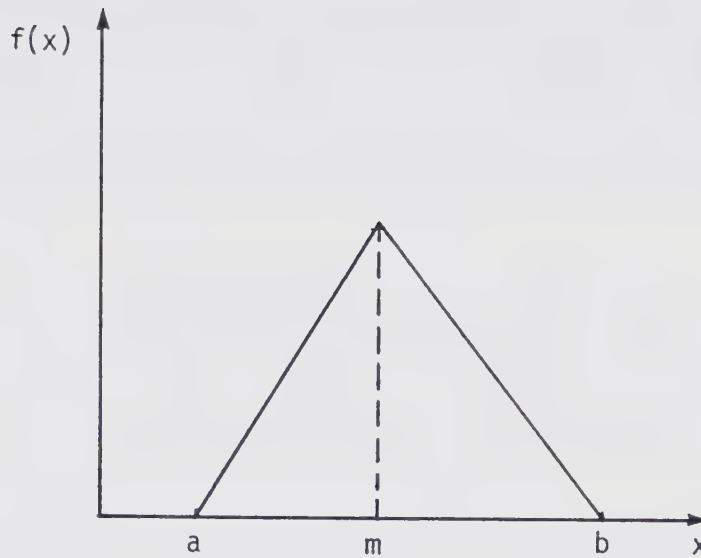
r_{ij} = the correlation coefficient for the gross margins of the i th and j th activities.

The relationships expressed by Equations 2.15 and 2.16 are used in the risk budgeting program to manipulate and summarize the subjective information provided by the farm manager.

The information required from the farm manager to estimate the expected gross margin and variance of the gross margin are his subjective estimates of prices, yields and variable costs. As well, the activity levels of the proposed enterprises in the operating plan are necessary. The subjective estimates are incorporated into the budgeting procedure through the triangular probability density function⁸ as displayed in Figure 2.15. The probability density function of the triangular distribution is expressed as

Figure 2.15

The Triangular Probability Distribution Function



$$\begin{aligned}
 f(x) &= 2(x-a)/(m-a)(b-a), \quad a \leq x \leq m \\
 &= 2(b-x)/(b-m)(b-a), \quad m \leq x \leq b \\
 &= 0, \text{ otherwise}
 \end{aligned}
 \tag{2.17}$$

where

a = the most pessimistic value,

m = the most likely value,

b = the most optimistic value,

x = the random variable.

With the farm manager's subjective estimates of these values, through the properties of the triangular probability distribution, the means and variances can be expressed as

$$\mu = (a+m+b)/3 \tag{2.18}$$

$$\sigma^2 = [(b-a)^2 - (m-a)(b-m)]/18 \tag{2.19}$$

These equations are used to obtain the values of the expected yields and prices and their variances, which are subsequently incorporated

into the equations deriving the enterprise level gross margin and variance. The enterprise level estimates are then extended to a farm level basis through Equations 2.15 and 2.16. The correlation coefficients are applied during the calculation of the variance of the total gross margin, and modify that variance according to the group of enterprises considered.

The total farm operation estimates correspond to the analysis of one operating plan. Further plans can be analyzed in this manner by providing subjective estimates of prices, yields, and variable costs, plus the enterprise activity levels proposed for the alternate plans. Each of the expected gross margin-variance results generated from the budgeting procedure represent the technical and economic feasibility constraints associated with that individual operation. The farm manager can then compare and contrast the alternatives with respect to his risk attitudes (human feasibility constraints) to arrive at the enterprise selection decision.

E. SUMMARY

The progression from economic theory to the risk budgeting routine indicates the rationale behind the enterprise selection decision. The partial budget is noted to be the link between theory and practical application. Modifications to the partial budget approach to accommodate the uncertainty case make it possible to incorporate decision factors that are difficult to quantify.

By incorporating the farm manager's own subjective estimates of prices, yields and variable costs, the risk budgeting program derives the expected gross margin and the variance of the gross margin for the operating plan. This operating plan, and others, can then be evaluated according to the manager's risk attitudes leading to the enterprise selection decision.

Considerable discussion is provided regarding the role and impact of the correlation coefficient. The following chapter examines the methods and requirements involved with obtaining a correlation matrix pertinent to the study area.

ENDNOTES

1. Risk and uncertainty have been defined by Knight (23, pg. 19) according to the perceptions of their respective probability distributions. However, for the purposes of this thesis, they are both to be considered as referring to a situation in which a subjective probability judgement can be made, and consequently, will be used interchangeably.

2. To clarify this manipulation, note that an intermediate step in the manipulation specifies (20, pg. 398-400):

$$\partial x_j / \partial x_s = (-\partial F / \partial x_s) / (\partial F / \partial x_j) = dx_j / dx_s < 0$$

and similarly

$$\partial y_i / \partial y_k = (-\partial F / \partial y_k) / (\partial F / \partial y_i) = dy_i / dy_k < 0$$

$$\partial y_i / \partial x_j = (-\partial F / \partial y_i) / (\partial F / \partial x_j) = dy_i / dx_j > 0$$

where

$$\partial F / \partial y_i > 0$$

$$\partial F / \partial x_j < 0$$

3. The sign convention employed in stating the factor-factor and product-product marginal conditions at times has a tendency to create confusion among students of economics. Intuitively, it can be seen that the slopes of the price lines and their respective isoquants or product transformation curves are negative. This is not always interpreted clearly from the mathematical statement of the marginal conditions. To clarify this convention, the following statements show, as an example, the expansion of the factor-factor case.

$$(-1)(dy_k / dy_i) = (-1)(-p_i / p_k)$$

and, therefore

$$-dy_k / dy_i = p_i / p_k$$

Each side of the equation is multiplied by -1 to remain logically consistent. The MRS is a negative ratio such that resulting calculations of each side will yield positive quantities.

4. Higher order moments are assumed to be zero. This assumption is supported by estimation problems of utility functions.
5. As opposed to the certainty case, diversification in this context is used as a means to reduce risk. It is assumed that diversification attempts to conform to technological and economic

constraints have been exhausted in the certainty case. For example, diversification for the purpose of making more efficient (and timely) use of available machinery capacity, labor hours, etc., has already occurred (rather than incurring possible additional costs through specialization).

6. This assumption is included to simplify the procedure. Variable costs could be considered as stochastic in the evaluation as follows:

$$E = \sum_{i=1}^m p_i y_i - \sum_{j=1}^n c_j x_j$$

$$V = \sum_{i,j=1}^{m+n} r_{ij} \sigma_i \sigma_j$$

7. A more general statement for variance, given prices and yields are not independent, is provided by Anderson, Dillon and Hardaker (4, pg. 172).
8. The triangular distribution is employed in the budgeting program, as opposed to a beta distribution, due to simplicity of application. Firstly, as noted by Bauer (6, pg. 129), the degree of estimation error arising from the use of either the triangular or the beta distribution is comparable yet the triangular distribution has a much simpler mathematical form. This facilitates its use in the nature of the microcomputer program proposed. Secondly, since the budgeting tool is intended for extension use, it is imperative that an understanding of its components be easily transferable to intermediate and end uses. An understanding of more complex functions may be difficult to impart and their estimation requires more detailed input which may result in resistance to the extension tool as a whole. It is understood that a degree of accuracy in estimation may be traded off to promote better acceptance and understanding of the extension tool.

III. DATA AND METHODOLOGY

There are three pieces of information necessary for the functioning of the risk budgeting program. Two of these, the expected gross margin and the variance of the gross margin, can be obtained from the farm manager's records and experience. The third, the correlation coefficient, cannot. Therefore, the second objective of this thesis is to develop a series of interaction components (correlation coefficients) of gross margins among an array of crop enterprises. Correlation coefficients provide an indication of relative association, or a measure of how the enterprise gross margins vary together. As indicated in Chapter II, the risk budgeting procedure requires these coefficients in order to derive the estimate for variance of the gross margin for the total farm operation. Since it is highly unlikely that the farm manager would be able to accurately establish the coefficients for the range of enterprise alternatives available to him, it is necessary for them to be provided from an external source.

Data gathered and methodology applied to obtain the series of correlation coefficients sought is the focus of this chapter. Discussion of the data entails its sources, available detail, and its conformation. The conformation of the data is closely linked to the methodology in terms of the variety of cropping enterprises available, and the aggregation of the data into regions within the study area. Methodological issues are concerned with illustrating the logic of the procedure to obtain the correlation coefficients, adapting the procedure to the conformation of the data, and describing the significance testing process.

A. DATA

SOURCES AND LIMITATIONS

In the development of a correlation matrix for the study area (Extension Region II), secondary data on yield and price of the major crops within the area are necessary. Yield data are compiled from the yield report files of the Alberta Hail and Crop Insurance Corporation (AHCIC).¹ The data consist of individual yield observations for a range of crops through the study area. Price series are formulated from a variety of published sources (noted later in the complete enterprise listing). The lack of complete series for all crops desired restricts the final array of crop enterprises and therefore, suitable proxies are employed to give the widest possible selection.²

CONFORMATION OF THE ENTERPRISE LIST

Crop yield data for the years 1969-1979 were made available on the basis of location (municipality) and crop enterprise. The yield series formulated spans only eleven years due to incompleteness in the historical information, aggregation of series, or specification difficulties. However, series of short duration are more desirable in that they reflect the contemporary yield and market characteristics and are more sensitive regarding the analysis of alternatives being undertaken (1, pg. 16). The general crop yield series gathered are as follows:

1. Spring Wheat
2. Durum Wheat
3. Oats
4. Barley
5. Spring Rye
6. Flaxseed
7. Rapeseed

Yield observations for each of these crop enterprises are organized by year and by municipality. The municipality, or sub-area listing for Extension Region II is as follows:

1. MD 31 - Foothills
2. MD 44 - Rockyview
3. County 16 - Wheatland
4. County 17 - Mountainview
5. MD 48 - Kneehill
6. MD 47 - Starland
7. Special Areas (including Special Area 2, Special Area 3 and MD 34- Acadia Valley).

These yield series are combined with relevant price series to obtain the gross revenue series necessary to develop the correlation matrix.

The combination of the price and yield series enables an expansion of the enterprise list by designating certain (within crop) price and yield combinations as proxies for similar crops. The assumption underlying the formulation of these proxies is that the within year variation in yields of the quality classes are highly correlated. The price series collected are:

1. #1 CWRS Wheat³
2. Cash Wheat (Calgary)⁴
3. #2 Amber Durm Wheat³
4. #1 CW Oats³
5. #1 Feed Oats³
6. Cash Oats (Calgary)⁴
7. #1 CW 2-Row Barley³
8. #1 Feed Barley³
9. Cash Barley (Calgary)⁴
10. Designated (#1 CW 2-Row) Barley³
11. #1 CW Spring Rye⁵
12. #1 CW Flaxseed⁵
13. #1 Can Rapeseed⁵

This listing of price series corresponds to the final listing of enterprise alternatives. The general variations in price from one sub-area to the next is assumed to be non-significant, i.e., price differences between areas are assumed to be constant amounts reflecting differences in cost due to proximity to markets, etc.. Consequently, only one price series is formulated for each enterprise throughout the study area.

B. DEVELOPMENT OF THE CORRELATION MATRIX

The development of the correlation matrix is comprised of three phases. Firstly, the gross revenue series are obtained through the data base. Use of these series is subject to specific assumptions. Secondly, the properly ordered gross revenue series go through a process of matrix manipulations to yield the original correlation matrix. Finally, coefficient groupings in the original correlation matrix are tested for significance.

ESTABLISHMENT OF GROSS REVENUE SERIES

The yield and price series are initially ordered, by sub-area, into crop enterprise groupings. Gross revenue series are obtained for each crop enterprise and sub-area by multiplication of the respective average annual yields with the relevant prices. The resulting gross revenue figures represent expected gross revenues and are subject to the assumptions of stochastic independence of prices and yields.

The gross revenue series are then "detrended" (i.e. the trend component is removed from the time series) through ordinary least squares procedures. The objective of detrending is to provide correlation coefficients exhibiting joint variability due to randomness. By excluding the trend component, influences of technological change, institutional factors, etc., are removed, allowing a standardized basis for comparing the seasonal, cyclical and irregular components among the enterprise series. The reasoning behind detrending is related to the time frame from which the series of correlation coefficients are established and the risk budgeting process is undertaken. The budgeting process involves planning from the present into the future, capturing the farm manager's subjective estimates of the trend component. Consequently, the trend component should be removed when establishing the correlation coefficients (from a historical series of data) to provide a standardized basis for analysis.

Once these series of gross revenue residuals are established, it is possible to proceed with the necessary matrix operations to obtain the original correlation matrix. Note that the budgeting procedure employs gross margins rather than gross revenues as a measure of returns to an enterprise. However, through the assumption of non-stochastic variable costs (refer to the discussion of Equations 2.12 and 2.13), the series of gross revenue residuals can be used in reference to gross margin residuals.

variance of the gross margin for each enterprise. The off-diagonal elements are the covariance terms, which are a measure of how the enterprises vary together (19, pg. 120).

The second phase of matrix operations involves a series of manipulations relating to Equation 3.2. Equation 3.2 expresses the correlation coefficient in terms of the covariance and standard

$$r_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} \quad (3.2)$$

where:

r_{ij} = the correlation coefficient for the i th and j th enterprises,

σ_{ij} = the covariance for the i th and j th enterprises, and

σ_i = the standard deviation of the gross margin for the i th enterprise

deviation. From the variance-covariance matrix, a diagonal matrix, $[A]$, is formed. Its elements are the inverse of the square root of the individual variance terms in the variance-covariance matrix. The original correlation matrix, $[r]$, is then derived as expressed in Equation 3.3. This derivation involves a pre and post-multiplication of the variance-covariance matrix by the diagonal

$$[r] = [A][\text{Var-Cov}][A] \quad (3.3)$$

matrix, $[A]$.

The correlation matrix contains elements reflecting the degree to which each enterprise is related to the others. The value of the correlation coefficient, r , can range from -1, indicating a perfect negative correlation, to +1, indicating a perfect positive correlation. As

well, any determinant selected from a subset of the matrix $[r]$ must be positive (4, pg. 37).

SIGNIFICANCE TESTING

The objective of the significance testing procedure is to establish the homogeneity of a group of correlation coefficients. The details of this procedure are as outlined by Steel and Torrie (25, pg. 180). As well as a discussion of the general testing procedure, this section outlines the conformation of the matrices (defining levels within the matrices), and attempts to clarify the term "significance" with regard to the correlation matrix.

The correlation matrix derived in the previous section is illustrated schematically in Figure 3.2. As the portions above and below the main diagonal are symmetrical, further discussions are in reference to the top half of the matrix only. The shaded, main diagonal sections in Figure 3.2 (later referred to as matrix sectors) represent the intra-regional interactions among crops. Within these shaded sectors are the correlations between crops for a given municipality. Only these intra-regional components are isolated for testing. Within the main diagonal sectors, an arbitrary division into crop-crop sub-sectors was done, as illustrated in Figure 3.3. The notation for crops in Figure 3.3 follows the ordering set out earlier in the chapter for the enterprise list. This arbitrary division of the matrix sectors into crop-crop interaction sub-sectors provides the initial, logical grouping for significance testing.

The purpose of the significance testing is to determine the validity of the estimates obtained from a statistical procedure. It attempts to establish a measurable degree of "sameness" or "difference" in terms of the probability of the magnitude of the estimates being related to chance or due to the influence of an external factor. The significance testing procedure starts with deriving a pooled (or average) coefficient for a sub-sector grouping. The pooled coefficient is the basis for calculating a test statistic which establishes if the deviations from the average are cumulatively large enough to indicate that one, or more, of the coefficients in the sub-sector grouping are different from the other. Alternatively, using the notion of a confidence interval, the procedure indicates for a given confidence level that (a) the deviations from the average are suitably small, suggesting that the coefficients in the sub-sector are, in essence, the same, or (b) the deviations from the average are large enough to suggest that some of the coefficients are different and should be removed from the sub-sector grouping. Subsequently, a "significant grouping" refers to a homogeneous set of correlation coefficients as described in (a) above (i.e., the coefficients are not significantly different from each other).

The general significance testing procedure, as described by Steel and Torrie, involves an iterative procedure of testing possible sub-groupings within a major grouping (eg. matrix sector). If the sub-groupings are found to be heterogeneous, then they are altered (by removing or rearranging coefficients within the major grouping) until all sub-grouping are found to be homogeneous. The initial step in the

testing procedure is to convert all of the coefficients in the matrix sectors to Z scores (Z_i') through Equation 3.4. This conversion assumes the correlation coefficients follow a "near-normal" distribution,

$$Z_i' = 0.5 \ln (1+r/1-r) \quad (3.4)$$

thereby conforming to the properties of the Z, or normal, distribution.

Then, within the sub-groupings, a pooled Z score (Z_w') is calculated through the expression,

$$Z_w' = \frac{\sum_{i=1}^n (n_i - 3) Z_i'}{\sum_{i=1}^n (n_i - 3)} \quad (3.5)$$

where the number of observations (n_i) corresponds to the number of average annual estimates of gross revenue for each crop enterprise (i.e., eleven). A χ^2 test statistic, shown as follows,

$$\chi^2 = \sum_{i=1}^n (n_i - 3) (Z_i' - Z_w')^2 \quad (3.6)$$

is used to establish the homogeneity of the sub-grouping, with the degrees of freedom (d.f.) established by

$$\text{d.f.} = (\text{no. of } Z_i' \text{ in the sub-group}) - 1 \quad (3.7)$$

If the deviations about the average (Z_w') are large enough to cause the test statistic to exceed the tabulated value (for the proper number of d.f. and a given significance level) then the sub-grouping is not homogeneous and a regrouping is necessary. Regrouping is carried out until all sub-groupings within the major group are found to be homogeneous. The final step of the procedure involves converting all of the pooled Z scores (Z_w'), representing homogeneous groupings, back to correlation

coefficients through the expression

$$r = e^{\frac{2Z'_w}{2}} - 1/e^{\frac{2Z'_w}{2}} + 1. \quad (3.8)$$

As a result of the pooling and testing procedures, the correlation coefficients in the same homogeneous sub-group can be considered identical and are all represented by their Z' converted through Equation 3.8 (25, pg. 181).

The methodology for significance testing outlined in this section illustrates the general procedure employed for establishing homogeneous groupings of correlation coefficients. Reference to specific groupings, alternative group arrangements, and so on, have yet to be discussed. In Chapter IV the details and results of the analyses are presented.

ENDNOTES

1. These files included solicited (regular) yield estimates from farmers not experiencing yield losses in addition to those claiming indemnities.
2. Over time, as the present series are expanded, broader and more detailed series should be available for use.
3. Canada Grains Council, Canadian Grains Industry Statistical Handbook, 1974-79 inclusive, Table 36.
4. Alberta Agriculture, Agricultural Statistics Yearbook, 1971-79 inclusive, Table 26.
5. Canada Grains Council, op. cit., Table 38.
6. The lead component of the matrix notation refers to the number of years in the Series (11), while the lag component is the product of the 13 crop alternatives by the 7 sub-areas.

IV. ANALYSIS AND RESULTS

The analyses in this chapter refer to testing for homogeneous groupings within the original correlation matrix. Two approaches are taken regarding the analysis of the original matrix, each approach attempting to satisfy a specific objective. The first approach deals with an analysis of crop enterprise differences within each municipality. The objective is to establish whether or not the correlation coefficients for similar crops (eg. within the barley grouping) are different from each other. The objective of the second approach is to reveal interregional similarities between specific enterprise categories. Each of these approaches to analyzing the original correlation matrix is discussed in the following sections.

The selection of a significance level for determining the homogeneity of coefficients within a subsector requires an assessment of the trade-offs among a few factors. The nature of the data base and the manipulations to which it is subjected, as well as the level of statistical strictness necessary (in consideration for the requirements of the project) are taken into account. Given the degree of aggregation in the data base, the averaging procedures applied to this information (in addition to the general nature of the price series incorporated), and the necessity to identify general trends, a significance level of 90% is selected to indicate any departure from homogeneity within a grouping.

The results, in terms of general patterns arising from the analyses, are discussed in conjunction with the detailing of the analytical

procedures. Otherwise, the final correlation matrices, for use in the budgeting procedure, are supplied in the appendices.

A. INTRA-REGIONAL ANALYSIS

The intra-regional analysis deals with establishing homogeneous groupings of crops within individual location (municipality) matrix sectors. The initial arbitrary divisions of the matrix sectors into sub-sectors are as displayed in Figure 3.3. The logical crop groupings are arranged such that correlations among similar crops (eg. all barley crops vs. all oat crops, or all wheat crops vs. the flax alternative) are tested for homogeneity. Discretion is used in terms of any large scale grouping over different types of crops. In other words, even though a combination of two crops (eg. spring rye and rapeseed vs. oats) may possibly result in homogeneous group, such groupings are avoided on an intuitive and logical basis.

The procedure described in Chapter III is followed whereby the individual Z scores (Z_i') are calculated, then pooled over the group (Z_w'), and finally assessed through the χ^2 test. If the group is found to be heterogeneous, it is rearranged according to a logical pattern deduced from observing similarities among the individual coefficients. The new groupings are then pooled and tested. This procedure is followed until homogeneity is established for all groups throughout the matrix sector. The resulting seven intra-regional crop-crop correlation matrices are presented in Appendix 1.

A few general patterns among coefficients are observed in the final intra-regional matrices. Many of the original submatrix divisions are noted to be homogeneous. The only subsectors through all seven intra-regional matrices, which are further divided, are the wheat-barley, oat-oat and oat-barley combinations. The remainder of the subsectors display homogeneity.

In a few cases, higher correlations are observed among cash market crop enterprises relative to Cash-Wheat Board coefficients. However, this pattern is not predominant over all subsectors, nor is it consistent through all matrix sectors.

A final observation, which may play an important role regarding the risk budgeting procedure, is the sign and magnitude of the majority of the correlation coefficients for the flax enterprise. In all cases, the correlation coefficients between the flax enterprises is either very small and positive, or negative and usually small as well. The implication of this result, when considering the budgeting procedure, is that it provides a more obvious option for diversification. With small positive, or negative coefficients, a reduction in variability is more easily attainable.

B. INTER-REGIONAL ANALYSIS

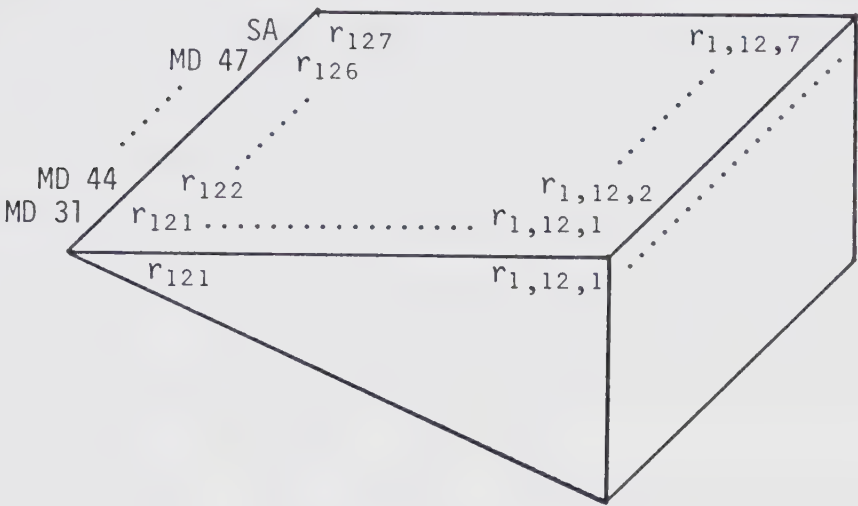
Inter-regional analysis is concerned with defining differences between individual crop enterprises, across the seven location matrices. The significance testing procedure starts with the regional matrix sectors from the original correlation matrix. The pooling

process is arranged in a manner such that the coefficients for an individual crop enterprise (eg. Cash Oats) are compared across the study area. Figure 4.1 provides a schematic representation of the groupings. Each group contains seven coefficients, that is, eg. the coefficients $r_{121}, r_{122}, \dots, r_{127}$ in Figure 4.1. The pooling and testing procedures are conducted as described in the methodology section. The final correlation matrix arising from the significance testing are presented in Appendix 2.

The results of the testing procedure bring forth one general pattern. All of the initial crop groupings are found to be homogeneous. This homogeneity indicates that the $7 \times 13 \times 13$ symmetrical matrix of original coefficients can be reduced to one 13×13 symmetrical matrix. Alternatively, when tested in this fashion, there is no significant difference between the regions within the study area, in terms of their correlation coefficients.

Further analysis on this matrix, to establish homogeneous crop groupings, is not performed. Application of the testing procedure described is somewhat experimental and the nature of the results from the two analyses arouses doubt as to the practicality of pursuing the analysis further. With significant divisions in the individual regional matrices as opposed to homogeneity across the general study area (on an individual crop basis), the usefulness and reliability of estimates received from further testing is questionable.

Figure 4.1
Schematic Representation of Inter-Regional Crop Groups



C. SUMMARY

Intra-regional analysis indicates homogeneous groupings at various levels of matrix divisions. The results of the inter-regional analysis show complete homogeneity across the study area for all individual crop enterprises. The patterns which become evident through these analyses may prove useful in initial planning of enterprise combinations. The intra-regional crop matrices satisfy the correlation coefficient requirements of the budgeting procedure. Chapter V includes discussion of the use of these matrices, as well as some further extension and applications of the budgeting procedure.

V. SUMMARY AND DISCUSSION

The objectives of this thesis are to (1) describe an enterprise selection budgeting procedure adapted to the evaluation of farm operating plans in a risky environment, and (2) develop a matrix of correlation coefficients for use in the budgeting procedure. The risk budgeting procedure is a farm planning tool designed to assist farm managers make better informed decisions. The risk budgeting program provides information such that the farm manager can assess alternative crop mixes given his attitudes toward risk.

In this chapter, the discussion is centered on the application of the risk budgeting program and expansion of the correlation matrix. The correlation matrix established offers a limited variety of crop enterprises. Expansion of the matrix through the introduction of additional crop enterprises and livestock enterprises would allow more complete farm planning efforts for those individuals whose enterprise alternatives are more diverse. It is to these farm planning efforts that the application of the risk budgeting program are directed.

A. APPLICATION OF THE RISK BUDGETING PROGRAM

Field application of the risk budgeting program as a micro-computer package is currently possible due to the efforts of G. Monner and the Airdrie Regional Office, Alberta Agriculture personnel, whose involvement entailed theoretical development and software design. In conjunction with these efforts, the development of the correlation

matrix is specifically directed to application in Alberta Agriculture Extension Region II. The correlation matrix, as presented, is an integral part of the extension tool.

The possible intermediate uses of the risk budgeting program are numerous. However, the major objective of the farm user is to establish a combination of enterprises which satisfy both his income/cash flow requirements and risk attitudes. The uses of the program range from establishing "bench mark" level of risk (through past and current operating plans), to rearranging enterprises and activity levels among commonly employed enterprises (within operating plans) to achieve a feasible solution, and as well, investigating diversification possibilities. The following discussion outlines the nature of the output from the risk budgeting program and its interpretations, and highlights some of its possible applications.

The output from the program, for an individual operating plan, provides a statement of the expected, or average gross margin and the variability associated with that return. The expected gross margin represents the returns above variable costs on the basis of a long term average, i.e., if the conditions leading to the estimate prevail for a number of years and the same operating plan is repeated, the average return over those years is given by the expected gross margin. Consequently, operating plans can be ranked in a relative sense, by their expected returns. However, the variability of these returns may elicit a subjective alteration of the operating plan ranking arising from risk attitudes. The variability estimates calculated by the program are stated as standard deviations (S.D.'s). The standard deviation

indicates the percentage frequency of which the returns will occur within a given range of the average. Therefore, the farm manager is able to evaluate (and rank¹) operating plans in terms of the expected returns and the variability of those returns.

This program provides a method which the farm manager can use to evaluate combinations of enterprises and activity levels. With the program, it is possible (for a limited set of alternatives) to (1) establish the sensitivity of the operating plan to changes in any of the variables supplied as estimates; (2) identify enterprises in terms of their relative risk and expected returns; and (3) evaluate small or large scale diversification attempts in order to reduce risk. It is obvious that there are a vast number of possible enterprise and activity level combinations which may require much time and effort to perform a complete analysis. If a complete analysis of available alternatives is desired, use of a more complex optimizing model may be in order. However, the farm manager may employ his preferences and knowledge of cultural practices, and then use the risk budgeting program in a step-wise, interval fashion to narrow down the number of alternatives. This process is not likely to yield the "optimal" solution but may put him in the neighborhood of that result.

In addition, since the budgeting program is intended as an extension tool, note that the intention is not to provide an optimal solution. It is more to provide a means by which the farm manager can perform a degree of analysis given a group of concepts that can be readily grasped and understood, and a model that is easily accessed with minimal input requirements. Admittedly, the number of alternatives that

could be considered are less than through quadratic programming or MOTAD, but effective extension techniques take a farmer in small steps. The risk budgeting program is a simple, understandable technique, offering credibility in technology transfer.

Consequently, by providing the farm manager with information on expected returns and variability for alternative operating plans, the complexity of the enterprise selection decision is reduced. The farm manager must then apply his risk attitudes (and knowledge of cash flow constraints associated with each plan) such that a decision can be made on the enterprise mix most likely to move him closer to his goal.

B. EXPANSION OF THE CORRELATION MATRIX

The present correlation matrix offers thirteen crop/marketing alternatives limited to the major crop enterprises in Extension Region II. Further expansion of the matrix could include crop enterprises of intermediate and subsidiary importance, and a range of livestock enterprises. The resulting matrix would give, in addition to location specific crop-crop interactions, location specific crop-livestock and livestock-livestock interactions for use in the budgeting procedure.

The initial stage of selection and definition of new enterprises involves the application of a priori knowledge of the type of enterprises. Sale dates, quality classes, weight classes, etc. may distinguish a proposed enterprise from others anticipated. However, the discerning characteristic of enterprises included in the analysis is that the pattern of variation of returns over time is observed to be

relatively dissimilar. For example, it would be expected that crops, as a group, would be more highly correlated than a crop-livestock comparison in terms of their gross revenue series over time. However, within a grouping of livestock enterprises (eg. beef) there may be, for different locations or different weight classes, enterprises which can be singled out on an a priori basis from the others. Consequently, to minimize redundancy in the analysis, enterprises which can be considered as similar in their returns should be introduced as a common grouping.

The procedure for expansion of the correlation matrix parallels that outlined in Chapter III. Price and yield data must be collected to obtain annual enterprise gross revenue estimates. The years of estimates must coincide with the existing series or, alternatively, the existing series could be updated to match the new series collected. The subsequent steps entail detrending, matrix manipulations, and significance testing as previously described.

C. A LOOK AHEAD: FURTHER RESEARCH AND DEVELOPMENT

Arising from the discussions of both the risk budgeting process and the correlation matrix, latitude for further research and application is readily apparent. The budgeting program could be expanded, for example, to include variability in major cost components. The correlation matrix could be expanded to include further (crop and livestock) enterprise alternatives, and as well, to provide more extensive coverage of the Province. As noted in Chapter IV, the significance

testing procedure, although adequate for the initial analysis intended, could benefit from further technical examination and determination of the full depth of its suitability. These types of suggestions are not uncommon for a study which can be essentially viewed as a "pilot project".

In addition to studies directly related to the risk budgeting program, possibilities exist to employ the program in aspects of behavioral research. The product of the program is expected income-variance pairs to which the farm manager applies his attitudes toward risk. Observation of enterprise selection decisions by farm managers from a relatively standardized set of alternatives may provide a means to gain further insight into risk attitudes, preferences and responses to risk.

Over the course of the discussions in this thesis, one overriding principle becomes apparent. Although considerable technical economic detail is involved in support of the risk budgeting process, the process itself is geared to a practical end use. For the farm manager to benefit from the process, it is not necessary that he understand all the technical detail. He needs only to understand the underlying concepts and principles.

ENDNOTES

1. A more objective ranking of operating plans can be attained by calculating the coefficient of variation (C.V.) as follows:

$$\text{C.V.} = \text{S.D.}/\text{E (GM)}$$

This effectively standardizes all operating plans such that the percentage variation about the mean can be compared among operating plans rather than through a subjective evaluation.

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Appendix 1

Intra-Regional Crop Matrices

Key: Codes

- W1. #1 CWRS Wheat (CWB)
- W2. Cash Wheat (Calgary)
- W3. #2 Amber Durum (CWB)
- O1. #1 CW Oats (CWB)
- O2. #1 Feed Oats (CWB)
- O3. Cash Oats (Calgary)
- B1. #1 CW 2-Row Barley (CWB)
- B2. #1 Feed Barley (CWB)
- B3. Cash Barley (Calgary)
- B4. Designated (#1 CW 2-Row) Barley (CWB)
- SR. #1 CW Spring Rye (Wpg. Commodity Exch.)
- F . #1 CW Flaxseed (Wpg. Commodity Exch.)
- RS. #1 Can. Rapeseed (Wpg. Commodity Exch.)

[illegible]

Appendix 2

Inter-Regional Crop Matrix

| | W1 | W2 | W3 | O1 | O2 | O3 | B1 | B2 | B3 | B4 | SR | F | RS |
|----|----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|-------|
| W1 | 1 | .9043 | .9085 | .9141 | .8708 | .8893 | .7222 | .8939 | .8611 | .8747 | .6597 | -.0851 | .5664 |
| W2 | 0 | 1 | .8769 | .9040 | .8888 | .9485 | .5887 | .8771 | .9463 | .9557 | .3488 | -.0559 | .4579 |
| W3 | 0 | 0 | 1 | .9267 | .9129 | .8866 | .7531 | .9332 | .9053 | .8941 | .4413 | -.1269 | .6225 |
| O1 | 0 | 0 | 0 | 1 | .9896 | .9288 | .7813 | .9749 | .9046 | .9197 | .5504 | .0351 | .7441 |
| O2 | 0 | 0 | 0 | 0 | 1 | .9052 | .7763 | .9736 | .8902 | .9072 | .4954 | .0370 | .7613 |
| O3 | 0 | 0 | 0 | 0 | 0 | 1 | .6900 | .8938 | .9839 | .9209 | .3536 | -.0273 | .5004 |
| B1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | .8127 | .6846 | .6507 | .4258 | -.1423 | .7533 |
| B2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | .9048 | .9257 | .4796 | -.0432 | .7759 |
| B3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | .9383 | .2559 | -.0791 | .4891 |
| B4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | .3195 | -.0360 | .5778 |
| SR | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | -.2069 | .4957 |
| F | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | .1664 |
| RS | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

Appendix 3

The Effect of the Correlation Coefficient on Expected Income-Risk Outcomes

In Chapter II, the relationship between the enterprise correlation coefficients and diversification is discussed. The degree to which the returns from the enterprises included in the operating plan are correlated (vary with respect to each other) can have a marked effect on the expected income-risk combinations obtained. Diversification of enterprises implies the selection of specific enterprises in such a manner that risk is decreased. The following discussion provides a detailed example of how the degree of correlation among enterprises can affect the expected income-risk outcomes for an operating plan.

The example employs a two enterprise case with means and variances given. For illustrative purposes, the correlation coefficients are altered, holding the means and variances constant. Anderson et al (4, pg. 193) put forward a series of relationships in support of this concept. Expected income and income variance are stated as,

$$E(Y) = y_i \mu_i + (Z - y_i) \mu_j$$

and

$$V(Y) = \sigma_i^2 y_i^2 + 2r_{ij} \sigma_i \sigma_j (Z - y_i) + \sigma_j^2 (Z - y_i)^2$$

where $E(Y)$ = expected income for the operating plan,

y_i = activity level of the i th activity,

y_j = $(Z - y_i)$ = activity level of the j th activity,

Z = total units of the i th and j th enterprises available for production,

μ_i = expected (net) return per unit of the i th enterprise,

$V(Y)$ = variance of the expected income for the operating plan,

σ_i^2 = variance of the returns per unit of the i th activity, and

r_{ij} = correlation coefficient for the i th and j th enterprises.

The variance equation is then manipulated to obtain the following:

(1) given a perfect positive correlation between i and j

$$(r_{ij} = +1),$$

$$V(Y)^{0.5} = \sigma_i y_i + \sigma_j (Z - y_i)$$

(2) given i and j are uncorrelated ($r_{ij} = 0$),

$$V(Y)^{0.5} = [\sigma_i^2 y_i^2 + \sigma_j^2 (Z - y_i)^2]^{0.5}$$

and

(3) given a perfect negative correlation between i and j

$$(r_{ij} = -1),$$

$$V(Y)^{0.5} = \sigma_i y_i - \sigma_j (Z - y_i).$$

These three relationships, plus the expected income equation, are employed to provide a numerical and graphical illustration. Given values for the variables as,

$$Z = 6$$

$$\mu_1 = 2$$

$$\sigma_1^2 = 4$$

$$\mu_2 = 4$$

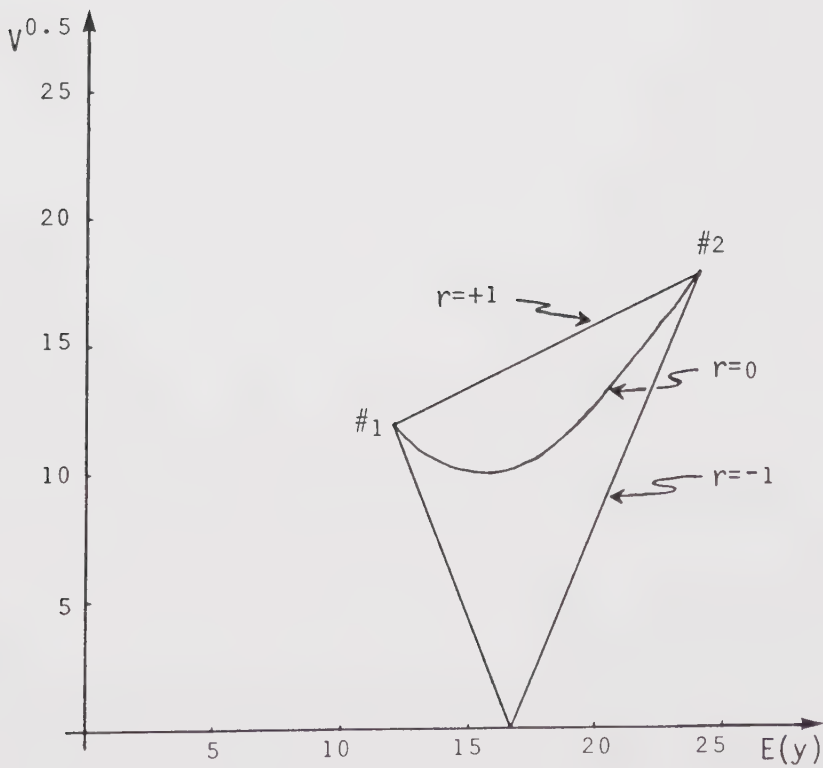
$$\sigma_2^2 = 9$$

the resulting expected income, $E(Y)$ and standard deviation, $V(Y)^{0.5}$, combinations are as follows:

| y | y | E(Y) | $V(Y)^{0.5}$ | | |
|---|---|------|--------------|-------|--------|
| | | | r = +1 | r = 0 | r = -1 |
| 0 | 6 | 24 | 18 | 18.0 | 18 |
| 1 | 5 | 22 | 17 | 15.1 | 13 |
| 2 | 4 | 20 | 16 | 12.6 | 8 |
| 3 | 3 | 18 | 15 | 10.8 | 3 |
| 4 | 2 | 16 | 14 | 10.0 | 2 |
| 5 | 1 | 14 | 13 | 10.4 | 7 |
| 6 | 0 | 12 | 12 | 12.0 | 12 |

These combinations are depicted graphically in Figure A.1.

FIGURE A.1: THE EFFECT OF CORRELATION ON EXPECTED INCOME-RISK OUTCOMES



In conjunction with the discussion in Chapter II, by envisaging a family of utility curves for a risk averse individual super-imposed upon Figure A.1, the optimal enterprise mix becomes apparent. This risk averse individual will tend to diversify his operating plan by including a mixture of negatively correlated enterprises in order to increase his expected utility.

Appendix 4

Sample Calculation Using the Risk Budgeting Procedure

As an illustration of the manner in which the risk budgeting procedure operates, a hypothetical case is presented. The following information constitutes a sample of the information necessary for an operating plan.

Enterprise 1: Wheat (fallow)

y = 125 acres = activity level of enterprise 1
c = \$54.00/acre = variable cost per acre of enterprise 1
(not including the value of operator labor)

Price and Yield Estimates

| | most pessimistic <u>(a)</u> | most likely <u>(m)</u> | most optimistic <u>(b)</u> |
|-------------------------------|-----------------------------------|------------------------------|----------------------------------|
| Price (\$/bu.) | 4.25 | 4.75 | 5.00 |
| Yield (bu/ac.) | 35 | 40 | 45 |
| Gross Margin per Acre(\$). | 94.75 | 136.00 | 171.00 |

Enterprise 2: Barley (stubble)

y = 125 acres = activity level of enterprise 2

c = \$58.00/acre = variable cost per acre of enterprise 2
(not including the value of operator labor)

Price and Yield Estimates

| | most pessimistic <u>(a)</u> | most likely <u>(m)</u> | most optimistic <u>(b)</u> |
|------------------------------|-----------------------------------|------------------------------|----------------------------------|
| Price (\$/bu.) | 2.00 | 2.30 | 2.50 |
| Yield (bu/ac.) | 35 | 40 | 50 |
| Gross Margin per Acre(\$) | 12.00 | 34.00 | 67.00 |

Enterprise 3: Flaxseed (fallow)

y = 50 acres = activity level of enterprise 3
c = \$45.00/acre = variable cost per acre of enterprise 3
(not including the value of operator labor)

Price and Yield Estimates

| | most pessimistic <u>(a)</u> | most likely <u>(m)</u> | most optimistic <u>(b)</u> |
|----------------|-----------------------------------|------------------------------|----------------------------------|
| Price (\$/bu.) | 6.25 | 7.00 | 7.50 |
| Yield (bu/ac.) | 15 | 20 | 25 |
| Gross Margin | | | |
| per Acre(\$) | 48.75 | 95.00 | 142.50 |

The correlation coefficients (as given in Appendix 2) are,

$$r_{12} = 0.7222 \quad (\text{Wheat} - \text{Barley})$$

$$r_{13} = 0.0851 \quad (\text{Wheat} - \text{Flax})$$

$$r_{23} = 0.1423 \quad (\text{Barley} - \text{Flax})$$

Enterprise expected gross margins and variances are derived through the budgeting procedure as follows:

$$\mu_i = (a + m + b)/3$$

$$\mu_1 = \frac{(94.75 + 136.00 + 171.00)}{3} = \$133.90/\text{ac.}$$

$$\mu_2 = \frac{(12.00 + 34.00 + 67.00)}{3} = \$37.70/\text{ac.}$$

$$\mu_3 = \frac{(48.75 + 95.00 + 142.50)}{3} = \$95.40/\text{ac.}$$

$$\sigma_1^2 = [(b - a) - (m - a)(b - m)]/18$$

$$\sigma_1^2 = \frac{[(171.00 - 94.75) - (136.00 - 94.75)(171.00 - 136.00)]}{18}$$

$$\sigma_1^2 = 242.80$$

$$(\sigma_1 = \$15.60/\text{ac.})$$

$$\sigma_2^2 = \frac{[(67.00 - 12.00) - (34.00 - 12.00)(67.00 - 34.00)]}{18}$$

$$\sigma_2^2 = 127.70$$

$$(\sigma_2 = \$11.30/\text{ac.})$$

$$\sigma_3^2 = \frac{[(142.50 - 48.75) - (95.00 - 48.75)(142.50 - 95.00)]}{18}$$

$$\sigma_3^2 = 366.20$$

$$(\sigma_3 = \$19.10/\text{ac.})$$

These enterprise estimates are then combined, accounting for the influences of correlation, to establish estimates of the expected gross margin for the operating plan, $E(GM_T)$, and the variance of the total gross margin, $V(GM_T)$, through the "cornerstone equations",

$$\begin{aligned} E(GM_T) &= \sum_{i=1}^n y_i \mu_i \\ &= (125 \times 133.90) + (125 \times 37.70) + (50 \times 95.40) \\ E(GM_T) &= \$26,220.00 \\ V(GM_T) &= \sum_{i=1}^n \sigma_i^2 y_i^2 + 2 \sum_{i=1}^n \sum_{i < j} r_{ij} y_i y_j \sigma_i \sigma_j \\ V(GM_T) &= 9,982,119.1 \\ [V(GM_T)]^{0.5} &= \text{Standard Deviation of the Total Gross Margin} \\ &= \$3,159.40 \end{aligned}$$

The results are summarized in the table below (including the coefficients of variation for illustrative purposes).

| <u>Summary Table - Hypothetical Example</u> | | | | |
|---|--------------------------------|----------|----------|---|
| | Enterprise (per acre basis) | | | Operating Plan (total farm basis) |
| | <u>1</u> | <u>2</u> | <u>3</u> | |
| Expected Gross Margin (\$) | 133.90 | 37.70 | 95.40 | 26,220.00 |
| Standard Deviation (\$) | 15.60 | 11.30 | 19.10 | 3,159.40 |
| Coefficient of Variation (%) | 11.6 | 30.0 | 20.0 | 12.0 |

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